Theoretical modelling of an infinitely wide rigid cylinder rotating over a grooved surface in hydrodynamic lubrication regime

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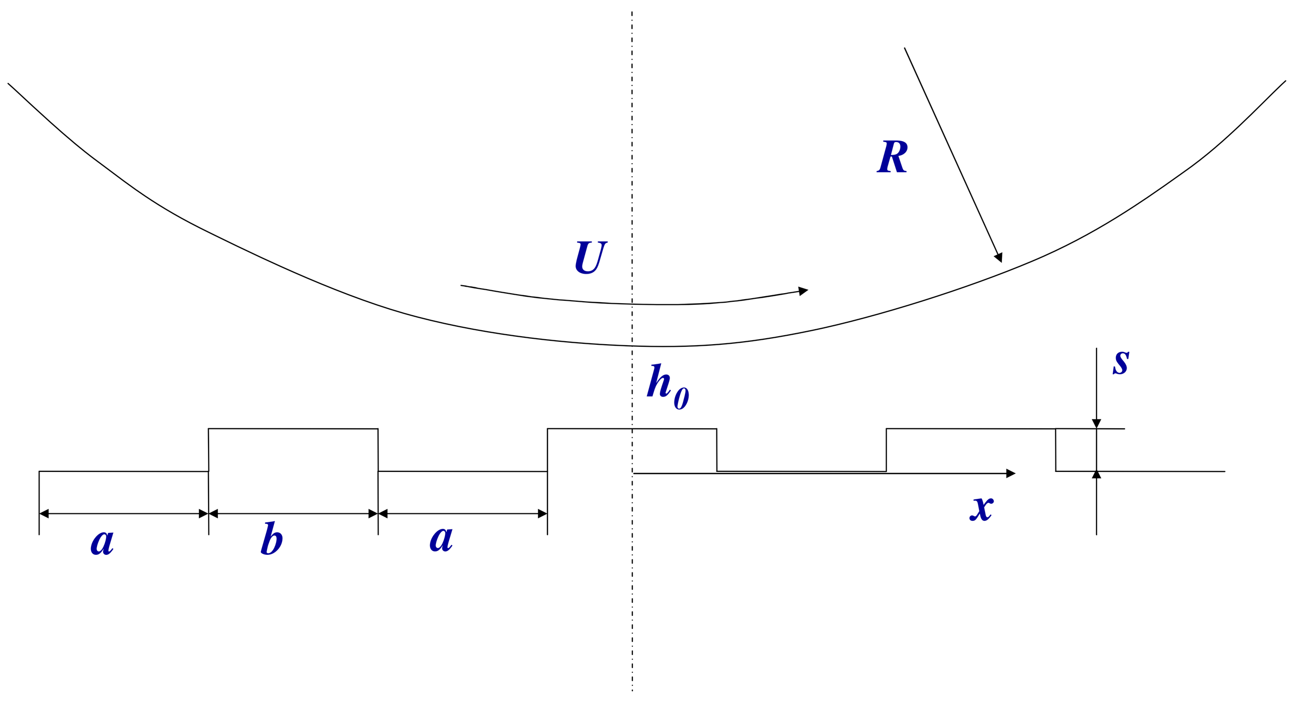
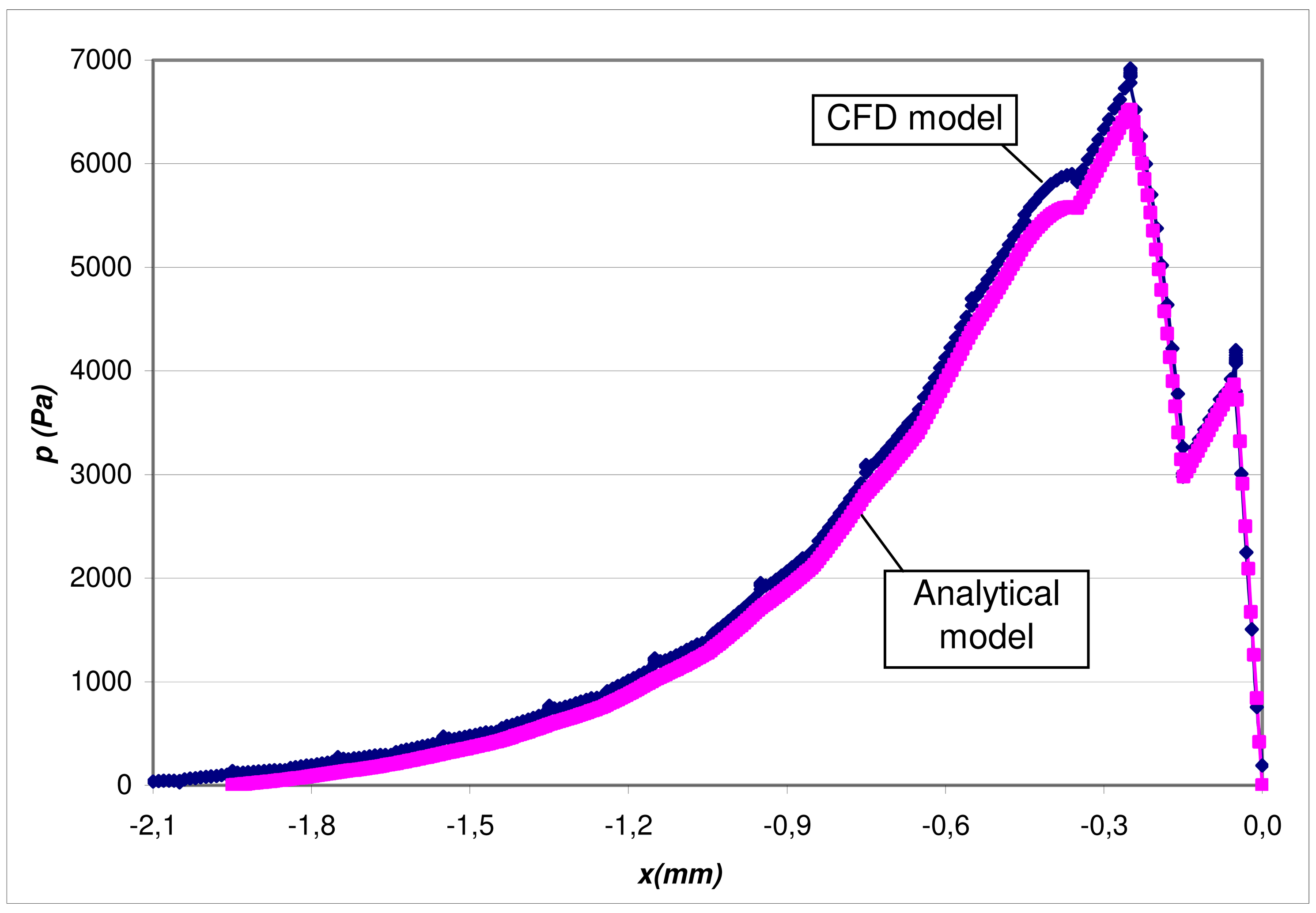
Schematic representation of the model

A theoretical model which describes an infinitely wide rigid cylinder rotating over a grooved surface in hydrodynamic lubrication regime is presented in this paper. Theoretical models of an infinitely wide cylinder rotating over a flat surface are presented by Cameron [1] and Hamrock [2].



Pressure distribution computation

The pressure distribution was also computed using a commercial CFD software. A comparison was made between the present analytical model and the numerical model.



Input data:
***R*=12.7mm, *U*=0.127m/s, *h₀*=10μm, *a*=100 μm, *b*=100 μm, *s*=10 μm, *n*=20 rot/min, *η*=0.02Pas.**

Governing Equations

The pressure is considered to be zero on the divergent zone. Using the parabolic approximation, the film thickness is calculated as follows:

$$h = h_0\left(1+\frac{x^2}{2Rh_0}\right)$$

Solving the Reynolds equation on a land surface the following relations are obtained:

$$\frac{dp}{dx} = 6\eta U\frac{h-\bar{h}}{h^3} \qquad \frac{1}{\cos^2\bar{\gamma}} = \frac{\bar{h}}{h_0}$$

$$\bar{p} = \frac{\gamma}{2}+\frac{\sin 2\gamma}{4}-\frac{1}{\cos^2\bar{\gamma}}\left[\frac{3\gamma}{8}+\frac{\sin 2\gamma}{4}+\frac{\sin 4\gamma}{32}\right]+C$$

On the groove surface the pressure distribution is:

$$\bar{p}_R = \left(\frac{h_0}{h_0+s}\right)^{3/2}\left(\frac{\gamma_R}{2}+\frac{\sin 2\gamma_R}{4}-\frac{1}{\cos^2\bar{\gamma}_R}\left[\frac{3\gamma_R}{8}+\frac{\sin 2\gamma_R}{4}+\frac{\sin 4\gamma_R}{32}\right]\right)+C$$

At the discontinuities the flow conservation is applied.

Load Carrying Capacity

$$\bar{F} = \sum_{i=1}^{n/2-1}\int_{\arctan\left(-\frac{b/2+a+(i-1)(a+b)}{\sqrt{2R(h_0+s)}}\right)}^{\arctan\left(-\frac{b/2+(i-1)(a+b)}{\sqrt{2R(h_0+s)}}\right)} \bar{p}_R(\gamma)\frac{1}{\cos^2\gamma}\sqrt{\frac{h_0+s}{h_0}}d\gamma + \sum_{i-1}^{n/2-1}\int_{\arctan\left(-\frac{b/2+a+b+(i-1)(a+b)}{\sqrt{2Rh_0}}\right)}^{\arctan\left(-\frac{b/2+a+(i-1)(a+b)}{\sqrt{2Rh_0}}\right)} \bar{p}_L(\gamma)\frac{1}{\cos^2\gamma}d\gamma + \int_{\arctan\left(-\frac{b/2}{\sqrt{2Rh_0}}\right)}^{0}\bar{p}_L(\gamma)\frac{1}{\cos^2\gamma}d\gamma$$

Friction force

$$\bar{F}_f = \sum_{i=1}^{n/2-1}\int_{-(b/2+a+(i-1)(a+b))}^{-(b/2+(i-1)(a+b))}\left(-3\frac{(h(x)+s)-\bar{h}}{(h(x)+s)^2}-\frac{1}{h(x)+s}\right)dx + \sum_{i=1}^{n/2-1}\int_{-(b/2+a+b+(i-1)(a+b))}^{-(b/2+a+(i-1)(a+b))}\left(-3\frac{h(x)-\bar{h}}{h(x)^2}-\frac{1}{h(x)}\right)dx + \int_{-b/2}^{0}\left(-3\frac{h(x)-\bar{h}}{h(x)^2}-\frac{1}{h(x)}\right)dx$$

Notations

- a – groove side
- b – land side
- D – cylinder diameter
- F – load carrying capacity
- $\bar{F}$ – dimensionless load carrying capacity, $\frac{Fh_0}{\eta UwR}$
- F_f – friction force
- $\bar{F}_f$ – dimensionless friction force, $\frac{F_f}{\eta Uw}$
- h – film thickness
- $\bar{h}$ – film height where the pressure gradient is zero
- h_0 – minimal film thickness
- n – total number of grooves and lands
- p – fluid pressure
- $\bar{p}$ – dimensionless pressure, $\frac{ph_0^2}{6\eta U\sqrt{2Rh_0}}$
- R – fluid pressure on the grooves (recesses)
- R – cylinder radius
- s – groove height
- U – cylinder speed
- w – cylinder width
- $\bar{x}$ – dimensionless x coordinate, $\frac{x}{\sqrt{2Rh_0}}$
- γ – substitution angle, $\arctan\bar{x}$
- γ_R – substitution angle on the grooves,
- η – fluid viscosity
- μ – friction coefficient
- $\bar{\mu}$ – dimensionless friction coefficient, $\mu\frac{R}{h_0}$

References

1. Cameron, A., The principles of lubrication, London, Longhmans, 1966
2. Hamrock, B.J. , Schmid, S. R., Jacobson, B. O., Fundamentals of fluid film lubrication, Marcel Dekker, 2004